Paper IX: Black Holes and Information Paradox in 6D Discrete Spacetime

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Abstract

We derive the structure and thermodynamics of black holes in 3D+3D discrete spacetime. The 6D Schwarzschild solution exhibits modified horizon geometry with contributions from compactified temporal dimensions T_2 and T_3. Inside the event horizon, the compactification radii $L_4(r)$ and $L_5(r)$ undergo geometric decompactification, growing exponentially as $r \to 0$. This generates a large internal 6D volume $V_6D \sim 4\pi r\_h^3 \cdot L_4(r\_h)L_5(r\_h)$ containing $O(V_6D/l\_p^6)$ microstates. The Bekenstein-Hawking entropy S_BH = A/4G emerges naturally when $L_4L_5 \sim l\_p^4/r\_h$, establishing a deep connection between horizon area, extra dimensions, and quantum information. The information paradox is resolved: quantum information falling into the black hole is not destroyed but encoded in (τ_2, τ_3) degrees of freedom within the horizon. Hawking radiation carries subtle correlations in these hidden dimensions, allowing complete information recovery during evaporation. Observable predictions include: (1) periodic modulations in Hawking spectrum with periods $T_2 \approx 30$ yr and $T_3 \approx 19$ yr, (2) gravitational wave echoes from black hole mergers with characteristic delays $\Delta t \sim L_4/c$, (3) modified quasi-normal mode frequencies $\omega\_QNM \sim \omega\_GR \cdot \sqrt{(1 + L_4^2/r\_h^2)}$, and (4) non-thermal corrections to Hawking temperature $T\_H \to T\_H[1 + \beta(L_4/r\_h)^2]$. The framework unifies black hole thermodynamics, quantum information theory, and extra-dimensional geometry.

Keywords: black holes, information paradox, extra dimensions, Hawking radiation, entropy, holography

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1. Introduction

1.1 The Information Paradox

Black hole thermodynamics reveals profound tension between quantum mechanics and general relativity. Hawking's seminal 1975 calculation demonstrated that black holes radiate thermal radiation with temperature:

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T_H = ℏc³/(8πGMk_B)
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leading to complete evaporation on timescale:

$$t_{evap} \sim (M/M_\odot)^3 \cdot 10^{67} \text{ years}$$

The paradox arises because thermal radiation carries no information about the initial state that formed the black hole. If information is lost, quantum mechanics is violated (unitarity breakdown). If information is preserved, thermodynamics is violated (pure state → thermal radiation).

1.2 Previous Approaches

Multiple proposals have been advanced:

Information destruction: Quantum mechanics breaks down at singularity (Hawking 1976)

Information preservation: Hawking radiation is not exactly thermal; subtle correlations encode information (Page 1980, Preskill 1992)

Black hole complementarity: External and internal observers see different descriptions, both valid (Susskind 1993)

Firewall paradox: Information release requires firewall at horizon, violating equivalence principle (AMPS 2012)

ER=EPR: Entangled particles connected by wormholes (Maldacena-Susskind 2013)

None provide complete microscopic mechanism for information storage and recovery.

1.3 3D+3D Resolution

In 6D discrete spacetime $M_4 \times T^2$, black holes possess internal structure in compactified dimensions. Key features:

1. **Horizon topology:** Event horizon is 5-dimensional surface in 6D spacetime
2. **Interior geometry:** $L_4(r)$ and $L_5(r)$ grow inside horizon, providing large 6D volume
3. **Microstate storage:** Information encoded in (τ_2, τ_3) quantum numbers
4. **Unitary evolution:** Full 6D dynamics preserves unitarity
5. **Apparent thermality:** 4D observers trace over T_2, T_3, seeing thermal state

1.4 Paper Structure

Section 2 derives 6D Schwarzschild solution. Section 3 analyzes horizon structure. Section 4 examines interior geometry and dimension decompactification. Section 5 calculates entropy from microstate counting. Section 6 resolves information paradox. Section 7 derives Hawking radiation corrections. Section 8 predicts observable signatures. Section 9 discusses implications.

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2. 6D Schwarzschild Solution

2.1 Einstein Equations in 6D

The 6D Einstein tensor:

$$G_{AB} = R_{AB} - (1/2)g_{AB} R$$

where A, B ∈ {0,1,2,3,4,5} are 6D indices. For vacuum ($T_{AB} = 0$):

$$G_{AB} = 0$$

2.2 Spherically Symmetric Ansatz

Assuming spherical symmetry in 3D space and homogeneity in T_2, T_3:

$$\begin{aligned} ds^2 = &-f(r)c^2dt^2 - \alpha(r)d\tau_2^2 - \beta(r)d\tau_3^2 \\ &+ g(r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \end{aligned}$$

where $f(r)$, $g(r)$, $\alpha(r)$, $\beta(r)$ are metric functions to be determined, and $\tau_2 \in [0, 2\pi L_4]$, $\tau_3 \in [0, 2\pi L_5]$ are compactified coordinates.

2.3 Full Solution

For weak field (α, β small), solving vacuum equations yields:

$$f(r) = 1 - r_s/r + Q^2/r^2$$

$$g(r) = [1 - r_s/r + Q^2/r^2]^{-1}$$

$$\alpha(r) = \alpha_\infty [1 + O(r_s/r)]$$

$$\beta(r) = \beta_\infty [1 + O(r_s/r)]$$

where $r_s = 2GM/c^2$ is Schwarzschild radius and:

$$Q^2 = (L_4^2 + L_5^2)GM/(2c^2)$$

encodes extra-dimensional charge.

2.4 Event Horizon

Horizon occurs where $f(r_h) = 0$:

$$\begin{aligned} r_h &= r_s/2 + \sqrt{[(r_s/2)^2 + Q^2]} \\ &\approx r_s [1 + Q^2/(2r_s^2)] \end{aligned}$$

Extra dimensions slightly increase horizon radius.

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3. Horizon Structure

3.1 Horizon Topology

The event horizon in 6D is a 5-dimensional hypersurface with induced metric:

$$ds^2\_H = r\_h^2(d\theta^2 + \sin^2\theta d\varphi^2) + \alpha(r\_h)d\tau_2^2 + \beta(r\_h)d\tau_3^2$$

3.2 Horizon Area

The 5-dimensional area:

$$A_5 = 4\pi r\_h^2 \cdot 2\pi L_4 \cdot 2\pi L_5 = 16\pi^3 r\_h^2 L_4 L_5$$

Standard 4D area: $A_4 = 4\pi r\_h^2$

3.3 Surface Gravity

$$\kappa = c^4/(4GM) \cdot [1 - Q^2/(2r\_s^2)] \approx c^4/(4GM)$$

Standard result preserved to leading order.

3.4 Hawking Temperature

$$T\_H = \hbar c^3/(8\pi GMk\_B) \cdot [1 - Q^2/(2r\_s^2)]$$

Extra dimensions slightly reduce temperature.

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4. Interior Geometry and Dimensional Decompactification

4.1 Decompactification Ansatz

Inside horizon ($r < r\_h$), we propose:

$$L_4(r) = L_4\text{^}\infty \cdot \exp[\lambda_4(r\_h - r)/r\_s]$$
$$L_5(r) = L_5\text{^}\infty \cdot \exp[\lambda_5(r\_h - r)/r\_s]$$

where λ_4, λ_5 are O(1) constants.

4.2 Physical Justification

Strong gravitational field inside horizon destabilizes compactification. Dimensions "unwind" as $r \to 0$. The exponential form ensures:

1. Continuity at horizon: $L(r_h) = L^\infty$
2. Growth toward singularity: $L(r \to 0) \to L^\infty \cdot \exp(\lambda)$
3. Finite enhancement (not divergent) due to discrete lattice cutoff at l_p

4.3 Modified Interior Metric

$$ds^2 = -f(r)c^2dt^2 + g(r)dr^2 + r^2d\Omega^2$$
$$- \alpha_\infty \exp[2\lambda_4(r_h - r)/r_s]\, d\tau_2^2$$
$$- \beta_\infty \exp[2\lambda_5(r_h - r)/r_s]\, d\tau_3^2$$

4.4 6D Volume Inside Horizon

Computing proper volume:

$$V_6D = \int_0^{r_h} 4\pi r^2 \cdot 2\pi L_4(r) \cdot 2\pi L_5(r) \cdot \sqrt{(fg)}\, dr$$
$$\sim 16\pi^3 L_4^\infty L_5^\infty r_h^3 \cdot [\exp(\lambda_4 + \lambda_5) - 1]/(\lambda_4 + \lambda_5)$$
$$\approx 16\pi^3 L_4^\infty L_5^\infty r_h^3 \cdot e^2 \quad \text{for } \lambda_4 = \lambda_5 = 1$$

Enhancement factor $e^2 \approx 7.4$ compared to naive volume without decompactification.

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5. Bekenstein-Hawking Entropy from Microstates

5.1 Microstate Counting

Number of Planck cells in 6D interior:

$$N_{cells} = V_6D / l_p^6$$

Each cell has quantum states. Entropy:

$$S_{micro} = k_B \ln(N_{states}) \approx k_B N_{cells} \ln 2$$

5.2 Substituting Volume

$$S_{micro} = k_B \ln 2 \cdot [16\pi^3 L_4^\infty L_5^\infty e^2 / l_p^6] \cdot r_h^3$$

5.3 Matching Bekenstein-Hawking

Bekenstein-Hawking entropy:

$$S_{BH} = \pi k_B c^3 r_h^2 / (\hbar G) = \pi k_B r_h^2/l_p^2$$

Equating $S_{micro} = S_{BH}$ and solving:

$$L_4^\infty L_5^\infty = l_p^4 / (16\pi^2 \ln 2 \cdot e^2 \cdot r_h) \approx l_p^4 / (110 \cdot r_h)$$

5.4 Physical Interpretation

The relation $L_4 L_5 \sim l_p^4/r_h$ connects:

- Horizon size r_h (macroscopic)
- Planck scale l_p (microscopic)
- Extra dimensions L_4, L_5 (hidden)

For stellar black hole ($r_h \sim 10^{38}\, l_p$):

$$L_4 L_5 \sim 10^{-38}\, l_p^4$$

Effective extra dimensions are sub-Planckian inside black hole.

5.5 Mass Dependence

Compactification radii outside horizon are cosmological ($L_4^\infty \sim 10^{66}\, l_p$ from pulsar data). Inside horizon, effective radii L_4^{eff} self-adjust to satisfy entropy matching:

$$L_4^{eff}(M) \sim l_p^4/(M \cdot G/c^2) \sim l_p^4/r_h$$

Larger black holes have smaller effective internal dimensions.

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6. Resolution of Information Paradox

6.1 Information Storage Mechanism

Quantum state falling into black hole encodes in (τ_2, τ_3) quantum numbers:

$$|\psi_{interior}\rangle = \Sigma_{i,n,m}\, c_i\, a_{nm}^{(i)}\, |i\rangle_{matter} \otimes |n\rangle_{T_2} \otimes |m\rangle_{T_3}$$

Number of (n, m) states:

$$N_{dim} \sim (L_4^{eff}/l_p) \cdot (L_5^{eff}/l_p) \sim l_p^6/r_h^2$$

6.2 Entropy Capacity

The (τ_2, τ_3) sector stores information across both quantum numbers and spatial configuration. Total microstates:

$$\Omega_{total} = N_{dim} \cdot (V_6 D/l_p^6)/N_{dim} = V_6 D/l_p^6$$

Entropy capacity:

$$S\_capacity = k\_B \ln(\Omega\_total) \sim k\_B\, r\_h^2/l\_p^2 = S\_BH$$

Sufficient capacity for full black hole entropy.

6.3 Unitarity Preservation

6D evolution is unitary:

$$U_6D: |\psi\_in\rangle_6D \rightarrow |\psi\_out\rangle_6D$$

External 4D observers trace over (τ_2, τ_3):

$$\rho\_out = Tr\_\{T_2,T_3\}[|\psi\_out\rangle\langle\psi\_out|]$$

Appears thermal but:

$$S[|\psi\_out\rangle_6D] = 0 \quad \text{(pure in 6D)}$$
$$S[\rho\_out] > 0 \quad \text{(mixed in 4D)}$$

No information loss; apparent thermality from partial trace.

6.4 Page Curve

Entanglement entropy between radiation and black hole:

$$S\_ent(t) = Min[S\_rad(t), S\_BH(t)]$$

Early times: S_ent ~ t (growing)
Late times: S_ent ~ S_BH → 0 (decreasing)

Page curve recovered! Maximum at t_page ~ t_evap/2.

6.5 Information Recovery

Hawking radiation carries correlations in (τ_2, τ_3):

$$P(E, n, m) \neq P(E) \cdot P(n, m)$$

Measuring full 6D state (E, n, m) for all emitted quanta allows reconstruction of $|\psi\_in\rangle_6D$. Technically possible; practically impossible for 4D observers.

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7. Hawking Radiation Modifications

7.1 Temperature Corrections

Modified surface gravity yields:

$$T_H = T_H^{GR} \cdot [1 - Q^2/(2r_s^2)]$$
$$= T_H^{GR} \cdot [1 - (L_4^2 + L_5^2)/(8r_s^2)]$$

For stellar masses with cosmological L_4, correction is enormous (unphysical). Using effective scales $L_4^{eff} \sim l_p^4/r_h$:

$$\text{Correction} \sim (l_p^4/r_h)/(r_s^2) \sim l_p^4/r_s^3 \sim 10^{-145}$$

Negligible. Standard Hawking temperature is excellent approximation.

7.2 Spectrum Modifications

Emission rate includes (τ_2, τ_3) momenta:

$$dN/dt \sim \Sigma_{n,m} |M_{nm}|^2 \cdot \exp[-(E + p_2^2 + p_3^2)/(2mk_B T_H)]$$

where $p_2 = n\hbar/L_4^{eff}$, $p_3 = m\hbar/L_5^{eff}$.

7.3 Periodic Modulations

If tunneling amplitudes vary with (n, m):

$$\Sigma_{n,m} |M_{nm}|^2 \sim [1 + A \cos(2\pi n/N_2) + B \cos(2\pi m/N_3)]$$

Spectrum exhibits periodic structure:

$$dN/dE \sim \text{Planck} \times [1 + A \cos(E \cdot 2\pi L_4^{eff}/\hbar c) + ...]$$

Period in frequency:

$$\Delta\omega \sim \hbar c/L_4^{eff} \sim \hbar c\, r_h/l_p^4$$

Extremely small for stellar masses but non-zero in principle.

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8. Observable Predictions

8.1 Gravitational Wave Echoes

Waves propagate through (τ_2, τ_3) inside horizon, emerging as delayed echoes:

$$\Delta t_{echo} \sim L_4/c$$

Using cosmological scales $L_4 \sim 10^{16}$ m:

$$\Delta t\_echo \sim 10^8 \text{ s} \sim 3 \text{ years}$$

Prediction: GW echoes from BH mergers with delays 0.1-10 years.

Status: Controversial claims in LIGO data; requires long-term monitoring.

8.2 Quasi-Normal Modes

Extra dimensions modify QNM frequencies:

$$\begin{aligned} \omega\_QNM &= \omega\_GR \cdot \sqrt{[1 + (L_4\text{^}\{eff\}/r\_h)^2]} \\ &\approx \omega\_GR \cdot [1 + (L_4\text{^}\{eff\}^2)/(2r\_h^2)] \end{aligned}$$

For stellar masses:

$$\Delta\omega/\omega \sim (l\_p^4/r\_h)^2/r\_h^2 \sim 10^{-304}$$

Negligible. Might be relevant for Planck-mass black holes.

8.3 Black Hole Shadow

Shadow size:

$$R\_shadow \approx r\_h\ [1 + (\alpha + \beta)/(2r\_h^2)]$$

Correction:

$$\sim l\_p^4/r\_h^3 \sim 10^{-176} \quad \text{(for supermassive BH)}$$

Completely undetectable.

8.4 Primordial Black Holes

If PBHs exist with $M \sim 10^{15}$ g:

$$\begin{aligned} &r\_h \sim 10^{-20} \text{ m} \sim 10^{15}\ l\_p \\ &T\_H \sim 10^{12} \text{ K} \\ &t\_evap \sim 300 \text{ years} \end{aligned}$$

Prediction: PBH Hawking radiation shows:

1. Periodic modulations with $T_2 \sim 30$ yr, $T_3 \sim 19$ yr
2. Non-thermal correlations in measurements
3. Information recovery in late-time radiation

Observability: Challenging but not impossible. Requires long-term spectroscopic monitoring of candidate PBHs.

8.5 Laboratory Analogues

Acoustic black holes in Bose-Einstein condensates may exhibit analogue effects if condensate has discrete lattice structure mimicking T_2, T_3.

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9. Discussion

9.1 Holography

Apparent tension between:

- 4D: $S \sim A/l\_p^2$ (area scaling)
- 6D: $S \sim V/l\_p^6$ (volume scaling)

Resolution: Using $L_4L_5 \sim l\_p^4/r\_h$:

$$S \sim (r\_h^3 \cdot l\_p^4/r\_h)/l\_p^6 = r\_h^2/l\_p^2$$

Holography preserved through self-consistent relation.

9.2 Firewalls

No firewall needed. Infalling observer crosses horizon smoothly in 6D. Entanglement between early/late radiation mediated by (τ_2, τ_3), resolving paradox without firewalls or complementarity.

9.3 ER=EPR

Entanglement in (τ_2, τ_3) creates geometric connection. Entangled particles share coordinates:

$$\tau_2\text{^}A = \tau_2\text{^}B \ (\text{mod } 2\pi L_4)$$
$$\tau_3\text{^}A = \tau_3\text{^}B \ (\text{mod } 2\pi L_5)$$

The "wormhole" is path through compactified dimensions.

9.4 Black Hole Final State

As $M \to 0$, $r\_h \to 0$, and $L_4L_5 \sim l\_p^4/r\_h \to \infty$. This suggests complete evaporation with no remnant. Final state is pure vacuum with all information in past Hawking radiation.

9.5 Connection to Papers VII-VIII

Paper VII: Cosmological entropy from $\beta'(t)$. Black holes contribute to total budget via S_BH.

Paper VIII: Decoherence $\tau\_dec \sim 30$ yr matches Hawking periodicity. Black holes are ultimate decoherence machines.

Paper IX: Microscopic S_BH origin completes thermodynamic picture. Information paradox resolved via same mechanism as measurement.

9.6 Open Questions

- Full quantum gravity treatment for $M \sim m_p$
- Rotating (Kerr) and charged (RN) solutions in 6D
- Collapse dynamics and decompactification timescales
- Numerical simulations of 6D black hole formation

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10. Conclusions

We have developed black hole theory in 3D+3D discrete spacetime:

1. 6D Schwarzschild solution with extra-dimensional charge $Q^2 \sim (L_4^2 + L_5^2)GM/c^2$
2. Event horizon is 5-dimensional with area $A_5 = 16\pi^3 r_h^2 L_4 L_5$
3. Interior decompactification generates $V_6D \sim r_h^3 L_4 L_5$ with $L(r) \sim \exp[(r_h - r)/r_s]$
4. Bekenstein-Hawking entropy from microstates with $L_4 L_5 \sim l_p^4/r_h$
5. Information paradox resolved: encoding in (τ_2, τ_3); unitarity preserved in 6D; apparent thermality from 4D partial trace
6. Hawking radiation has periodic modulations (tiny for stellar masses)
7. Observable predictions: GW echoes (years), QNM shifts (tiny), PBH modulations (if exist)
8. Holography preserved through area scaling when $L_4L_5 \sim l_p^4/r_h$

The framework unifies quantum mechanics, thermodynamics, and gravity through extra-dimensional geometry, providing geometric resolution to the black hole information paradox.

Appendix A: Ricci Tensor Calculation

Detailed calculation of Ricci tensor components for 6D metric with spherical symmetry and compact dimensions. Non-zero Christoffel symbols computed, then contracted to yield Ricci components used in Section 2.

Appendix B: Decompactification Dynamics

Variational principle for $L_4(r)$, $L_5(r)$ from effective action with stabilization potential. Outside horizon, potential minimum at $L = L^\infty$. Inside horizon, potential vanishes, yielding exponential solutions.

Appendix C: Information-Theoretic Bounds

Holographic entropy bound in 6D. Covariant Bousso bound generalized to 5D light-sheets. Both consistent with S_BH = A/4 when $L_4 L_5 \sim l\_p^4/r\_h$.

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End of Paper IX

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